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Superfluidity of indirect excitons and biexcitons in coupled quantum wells and superlattices

Yurii E Lozovik¹, Oleg L Berman^{1,2} and Magnus Willander³

¹ Institute of Spectroscopy, Russian Academy of Sciences, Troitsk, 142092, Moscow region, Russia

² City College of the CUNY, Convent Av. and 138th St., New York, NY 10031, USA

³ Goteborg University, Chalmers University of Technology, S-41296, Goteborg, Sweden

E-mail: lozovik@isan.troitsk.ru

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Abstract

The collective properties of indirect excitons in coupled quantum wells (CQWs) are considered. The energy of the ground state of the exciton *liquid* as a function of the density of electrons e and holes h at different separations D between e and h layers is analysed. The quantum gas–liquid transition as D decreases is studied. The superfluidity appearance temperatures in the system (Kosterlitz–Thouless transition temperatures) have been estimated at different separations D between e and h layers. For the anisotropic two-dimensional e – h system in CQWs the Mott metal–insulator quantum transition is considered. The instability of the ground state of the system of interacting two-dimensional indirect excitons in a slab of superlattice with alternating e and h layers is established. The stable system of indirect quasi-two-dimensional biexcitons, consisting of indirect excitons with opposite directed dipole moments, is considered. The radius and the binding energy of the indirect biexciton are calculated. The collective spectrum of a rare system of two-dimensional indirect biexcitons, interacting as electrical quadrupoles, is studied. The density of the superfluid component $n_s(T)$ and the Kosterlitz–Thouless phase transition to the superfluid state in this system are analysed.

1. Introduction

Two-layer systems of spatially separated electrons (e) and holes (h) in coupled quantum wells (CQWs) attracted experimental interest [1–5] especially in connection with the search for the superfluidity predicted for these systems, which can manifest itself as opposite persistent electric currents in each quantum well [6–11]. A set of other phenomena in the system is also interesting, particularly Josephson-type effects [6, 12, 13] (other interesting physics due

to tunnelling in two-layer systems was discussed recently in [14]) and properties of the e–h system in a single or coupled quantum well in strong magnetic fields [15–21].

One of the essential properties of spatially separated electrons and holes created by laser pumping is suppression of e–h recombination due to small overlap of electron and hole wavefunctions. Due to this fact the quasiequilibrium state of the system is easily attainable (at timescales $\tau > \tau_r \ll \tau_{rec}$, where τ_r and τ_{rec} are relaxation and recombination times, respectively). This gives the possibility of studying different equilibrium phases of the e–h system. Besides, an equilibrium spatially separated e–h system can be created by special electronic band engineering (see [6, 8, 9]).

The object of the present work is the study of properties of the excitonic *liquid* phase in the system of spatially separated electrons (e) and holes (h) in a CQW. The energy of the ground state E of the exciton liquid as a function of the density of electrons and holes n at different values of distance D between e and h layers is obtained.

From the analysis of the function $E(n, D)$ we investigate the quantum gas–liquid transition in the system of excitons with spatially separated electrons and holes.

We also study the Kosterlitz–Thouless transition of the exciton system in the superfluid state and the Mott transition from the excitonic (insulator) phase in an anisotropic e–h system to the electron–hole (metal) phase.

The problem of essential interest is also collective properties of excitons in superlattices and layered systems (see, e.g., [22]). One of the main goals of the present paper is also the study of the superfluidity of the rare electron–hole system in superlattices with alternating e and h layers. If ‘electron’ and ‘hole’ quantum wells alternate, at low densities excitons with spatially separated electrons and holes (indirect excitons) are created. Excitons in the same pair of quantum wells have parallel dipole moments, but dipole moments of excitons in neighbouring pairs of wells have opposite direction. This fact leads to essential distinction of properties of the e–h system in superlattices from that for isolated CQWs with spatially separated electrons and holes. In the latter case the indirect exciton system is stable due to dipole–dipole repulsion. In contrast to this it turns out that the exciton system in the slab of a superlattice with alternating e and h layers is unstable. This difference manifests itself even in a three-layer e–h–e or h–e–h system.

The paper is organized in the following way. In section 2 we calculate the energy of a CQW system in the Hartree–Fock approximation. In section 3 the correlation energy for the CQWs is analysed. In section 4 we describe the variational calculations for the CQWs. In section 5 we study the rare system of indirect excitons in CQWs. In section 6 the gas–liquid transition in the system of excitons with spatially separated electrons and holes in CQWs is investigated. In section 7 we study the Kosterlitz–Thouless transition of the system of the indirect excitons in CQWs in the superfluid state. In section 8 the Mott transition from the excitonic (insulator) phase in an anisotropic e–h system to the electron–hole metal phase is studied. In section 9 the instability of the ground state of the system of interacting indirect excitons in an isotropic slab of superlattice with alternating e and h layers is established. The system of two-dimensional indirect biexcitons, consisting of the indirect excitons in neighbouring pairs of wells, is stable. The radius and the binding energy of the indirect biexcitons are calculated. These biexcitons repel as electrical quadrupoles at long distances. In the ladder approximation the collective spectrum of the two-dimensional indirect biexcitons interacting weakly by the quadrupole law is considered.

The superfluid density $n_s(T)$ of interacting two-dimensional indirect biexcitons in superlattices is calculated at low temperatures T . The temperature of the Kosterlitz–Thouless phase transition [23] of the biexciton system to the superfluid state is calculated. In section 10 we discuss results of our work.

2. The Hamiltonian of the system. The Hartree–Fock approximation

To determine the conditions of existence (at temperature $T = 0$) of the exciton liquid state in the system with spatially separated e and h, we calculate the dependences of the energy of the ground state E on the (nonequilibrium) density n and calculate the minimum in the dependence $E(n)$ at different distances D between quantum wells.

The Hamiltonian of the system under consideration has the form

$$\hat{H} = \sum_{p=0}^{\infty} \left[\left(\frac{p^2}{2m_e} - \mu_e \right) a_p^+ a_p + \left(\frac{p^2}{2m_h} - \mu_h \right) b_p^+ b_p \right] + \frac{1}{2} \sum_{pp'k} \{ V(k) [a_p^+ a_{p'}^+ a_{p'+k} a_{p-k} + b_p^+ b_{p'}^+ b_{p'+k} b_{p-k}] - 2\tilde{V}(k) a_p^+ b_{p'}^+ b_{p'+k} a_{p-k} \}, \quad (1)$$

where a_p^+ and b_p^+ are electron and hole creation operators; m_e and m_h are effective masses of the electron and hole; $V(k) = \frac{2\pi e^2}{\epsilon k}$ is the Coulomb interaction in one layer; $\tilde{V}(k) = \frac{2\pi e^2}{\epsilon k} e^{-kD}$ is the interaction between electrons and holes in different layers; D is the distance between layers; ϵ is the static dielectric constant and μ_e and μ_h are chemical potentials, determined by conditions of particle conservation,

$$\sum_p \langle a_p^+ a_p \rangle = \sum_p \langle b_p^+ b_p \rangle = \frac{1}{2} N,$$

where N is the number of quasiparticles; $n = \frac{N}{S}$ is the surface density of quasiparticles and S is the area of the system.

For the calculation of the ground state energy of the exciton liquid of spatially separated e and h we consider pairing between electrons and holes [6, 24–26]. We use Gor'kov normal and anomalous Green functions [27, 28]:

$$\begin{aligned} G_{\alpha\beta}(x, x') &= -i \langle T(\tilde{\psi}_\alpha(x) \tilde{\psi}_\beta^+(x')) \rangle \\ F_{\alpha\beta}(x - x') &= e^{i\mu t} \langle N | T(\tilde{\psi}_\alpha(x) \tilde{\psi}_\beta(x')) | N + 2 \rangle \\ F_{\alpha\beta}^+(x - x') &= e^{-i\mu t} \langle N + 2 | T(\tilde{\psi}_\alpha^+(x) \tilde{\psi}_\beta^+(x')) | N \rangle, \end{aligned}$$

where $G_{\alpha\beta}(x, x')$ and $F_{\alpha\beta}(x - x')$ are the normal and anomalous Green functions, respectively, and $\tilde{\psi}_\alpha(x)$ and $\tilde{\psi}_\alpha^+(x)$ are one-particle Fermi operators; $\mu = \mu_e + \mu_h$.

At the beginning, let us consider the Hartree–Fock approximation as a zero approximation. In the uniform system diagrams of the Hartree approximation give zero contribution due to the electrical neutrality. Let us consider the exchange diagrams in the Hartree–Fock approximation.

Let us solve the equations for the normal and anomalous Green functions in the general case without the ordinary assumption for weak interaction (as in the BCS approximation) that the characteristic momenta of the quasiparticles are small in comparison with the Fermi momentum. The normal and anomalous Green functions in the Hartree–Fock approximation (with possible spontaneously broken symmetry $\Delta \neq 0$) are the solutions of the equations for the Gor'kov–Nambu (\hat{G}) matrix function:

$$\begin{aligned} \hat{G} &= \begin{pmatrix} G & F^+ \\ -F & G \end{pmatrix}, \\ G(p) &= \frac{\omega + \xi}{\omega^2 - \epsilon^2(p)}, \\ F^+(p) &= \frac{|\Delta(p)|^2}{\omega^2 - \epsilon^2(p)}. \end{aligned} \quad (2)$$

Here $\xi = \frac{p^2}{2m} - \mu$, $m = \frac{m_e m_h}{m_e + m_h}$ and

$$\epsilon(p) = \frac{1}{z_p^2 + 1} \left[(z_p^2 - 1)(p^2 - r^2 \mu p_0^2) - \frac{\sqrt{2} p_0 r}{\sqrt{\pi}} \int_0^\infty dp' \right. \\ \left. \times \int_0^{2\pi} d\phi \frac{(z_p^2 - 12z_p z_{p'} \exp[-D p_F \sqrt{p^2 - 2pp' \cos \phi + p'^2}]) p'}{(z_{p'}^2 + 1) \sqrt{p^2 - 2pp' \cos \phi + p'^2}} \right] \frac{p_F^2}{4}, \quad (3)$$

where $z_p = \frac{u_p}{v_p}$, and functions u_p^2 and v_p^2 are determined by the relations

$$u_p^2 = \frac{1}{2} \left(1 + \frac{\xi}{\epsilon(p)} \right); \quad v_p^2 = \frac{1}{2} \left(1 - \frac{\xi}{\epsilon(p)} \right)$$

where $z_p = \frac{u_p}{v_p} = \text{ctg} \phi_p$; $p_0^2 = 2 \int_0^\infty \frac{p dp}{1+z_p^2}$; all momenta are expressed in units $\frac{p_F}{p_0}$; p_F is the Fermi momentum; $r_s = \frac{1}{\sqrt{\pi n}}$; $\mu = \mu_e + \mu_h$ is the chemical potential of the e-h pair (see also [24, 25]). We introduce notions $p_0^2 = 2 \int_0^\infty \frac{p dp}{1+z_p^2}$; $p_F = (2\pi \hbar^2 n)^{\frac{1}{2}}$. We use units $\hbar = 1$, $m = 1$, $\frac{e^2}{\epsilon} = 1$.

In the Hartree-Fock approximation the gap Δ is determined by the self-consistency condition. Contrary to this in the used approximation for the intermediate type of binding the gap is determined by the variational calculation for total energy taking into account the correlation energy. As a probe function we use $z_p = u_p/v_p$ (see above).

The energy in the Hartree-Fock approximation can be expressed as

$$E = -Sp \int_0^g \frac{dg_1}{g_1} \int_0^\infty \frac{d\omega}{2\pi} \int \frac{d^2 p}{(2\pi)^2} [\hat{G}^{(0)}(\omega, p)]^{-1} [\hat{G}(\omega, p) - \hat{G}^{(0)}(\omega, p)] e^{i\omega t}, \quad (4)$$

where $t \rightarrow +0$; $\hat{G}^{(0)}(\omega, p) = G^{(0)}(\omega, p) \hat{1}$ is the Green function of the free particle; $\hat{1}$ is a unit matrix.

In the Hartree-Fock approximation the Green function \hat{G} is determined by equation (2). Accounting for equations (4) and (2), (3) we obtain the functional $E_{HF}\{n\}$ in the Hartree-Fock approximation:

$$2(E_{HF}\{n\}/n + \mu) = \frac{4}{r^2 p_0^4} \int_0^\infty \frac{p^3 dp}{1+z_p^2} \\ - \frac{\sqrt{2}}{\pi^2 r^2 p^3} \int_0^\infty p q dp dq \int_0^{2\pi} \frac{|V(p-q) + \tilde{V}(p-q) z_p z_q| d\phi}{(1+z_p^2)(1+z_q^2)}. \quad (5)$$

At high densities $na^{*2} \gg 1$ the gap Δ_p may be determined approximately as a result of the minimization of the Hartree-Fock functional $U\{z_p\}$. This gives, e.g. at $D \gg a^*$ (see [6]),

$$\Delta_p = \exp \left[-16D^2 p_0 \frac{1}{\pi a^*} \right], \quad (6)$$

where $a^* = \frac{\hbar^2 \epsilon}{4me^2}$ is the effective exciton Bohr radius.

But at small $D \lesssim a^*$ correlation effects are essential.

3. Correlation effects

We consider the excitonic phase in the Hartree-Fock approximation (with the gap $\Delta \neq 0$) as the initial zeroth order approximation for the ground state of the e-h system. Now we take into account essential diagrams for the correlation energy.

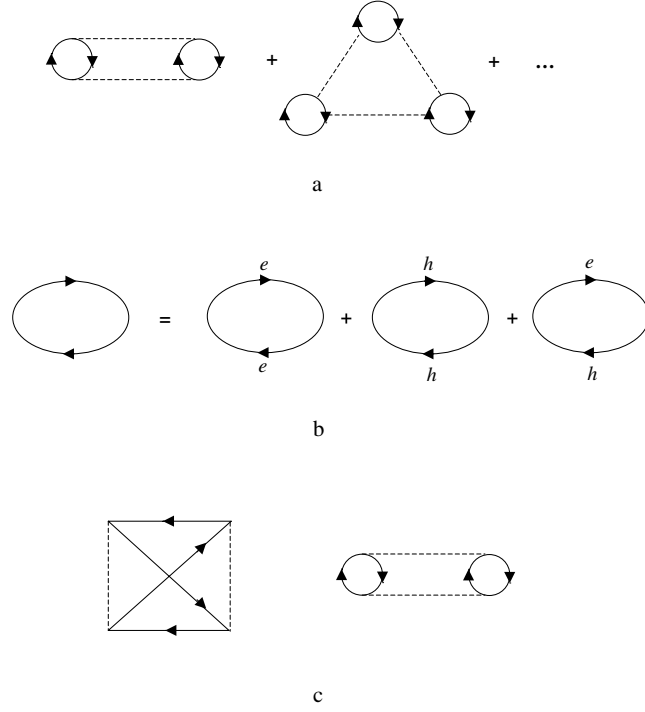


Figure 1. Diagrams for the correlation energy of the e–h system. (a) RPA diagrams, essential at $k/k_F \ll 1$; (b) polarization operator of the e–h system in RPA; (c) diagrams of second order taken into account at $k/k_F \gg 1$.

For small transferred momenta we will estimate the ratio of the minimal transferred momentum $k < \frac{\Delta}{v_F}$ to the Fermi radius p_F (at the equilibrium density n). The gap at the equilibrium density is the maximal one at the distance between layers $D = 0$. The variational calculations (see below) give

$$\frac{\Delta}{v_F p_F} = \frac{m\Delta}{2\pi\hbar^2 n} \approx 0.07 \ll 1. \quad (7)$$

So RPA diagrams [29] (figure 1(a)) prevail for small transferred momenta k ($x = \frac{k}{p_F} \ll 1$) not only for the metal phase, but also for the excitonic phase.

The correlation energy has the form [29]

$$E_1^c = -\frac{1}{n} \int \frac{d^2k}{(2\pi)^2} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} [\ln(1 - V_k \Pi(k, \omega)) - V_k \Pi(k, \omega)], \quad (8)$$

where $\Pi(k, \omega)$ is the polarization operator of the e–h system with account of e–h pairing (see figure 1(b)):

$$\Pi(k, \omega) = \Pi_{ee}(k, \omega) + \Pi_{hh}(k, \omega) + \Pi_{eh}(k, \omega). \quad (9)$$

For e–e and h–h polarization operators we have

$$\begin{aligned} \Pi_{ee(hh)}(k, \omega) = & -2 \int \frac{d^2p}{(2\pi)^2} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi i} [(G_{HF}(p, \epsilon)G_{HF}(-p+k, -\epsilon+\omega) \\ & + G_{HF}(p, \epsilon)G_{HF}(-p-k, -\epsilon-\omega) \\ & + (F_{HF}(p, \epsilon)F_{HF}(p+k, \epsilon+\omega) + F_{HF}(p, \epsilon)F_{HF}(p-k, \epsilon-\omega))], \end{aligned} \quad (10)$$

where G_{HF} and F_{HF} are normal and anomalous Green functions in the Hartree–Fock approximation equation (2).

$$\begin{aligned} \Pi_{eh}(k, \omega) = & -2 \int \frac{d^2p}{(2\pi)^2} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi i} (F_{HF}(p, \epsilon) F_{HF}(p+k, \epsilon+\omega) \\ & + F_{HF}(p, \epsilon) F_{HF}(p-k, \epsilon-\omega)). \end{aligned} \quad (11)$$

Without e–h pairing $\Pi_{ee} = \Pi_{hh} = \Pi_0$; $\Pi_{eh} = 0$, where Π_0 is the polarization operator for a normal *one-component* Fermi-gas.

Using equations (8)–(11) we find the correlation energy E_1^c for the small transferred momentum approximation ($x = \frac{k}{p_F} \ll 1$) as a functional of z_p .

The correlation energy E_2^c for large transferred momenta ($x = \frac{k}{p_F} \gg 1$) as a functional of z_p may be obtained as the sum of loop and exchange diagrams of the second order (figure 1(c)).

We used for the correlation energy E_{corr} of the system under consideration the interpolation between large and small momenta (as used in [30] for metallic e–h liquid, see also [24, 25]):

$$E_{corr} = \int_0^\infty q dq I(q), \quad (12)$$

where

$$I(q) = \frac{I_1(q)I_2(q)}{I_1(q) + I_2(q)}; \quad E_1^c = \int q dq I_1(q); \quad E_2^c = \int q dq I_2(q). \quad (13)$$

As a result taking into account equations (5) and (8) we obtain the total energy of the ground state E depending on parameters D and n :

$$E = E_{HF} + E_{corr}, \quad (14)$$

as the functional of u_p and v_p or the functional of $z_p = \frac{u_p}{v_p} = \text{ctg}\phi_p$ (we have $|v_p| = \frac{1}{\sqrt{1+z_p^2}}$, $|u_p| = \frac{|z_p|}{\sqrt{1+z_p^2}}$).

4. The variational calculations

To obtain the total energy of the ground state the minimization of $E(z_p)$ is performed numerically using probe functions z_p in the form

$$z_p = A \left(1 + \frac{p^2}{4} \right)^{\frac{3}{2}} + B, \quad (15)$$

where A and B are variational parameters (see also [25] where a two-dimensional excitonic liquid was calculated for one quantum well, $D = 0$).

The function (15) was chosen so that if $B = 0$ the function z_p^{-1} is equal to the Fourier transformation of the wavefunction of the two-dimensional Wannier–Mott exciton. For the semimetal state we have $v_p = 0$, and so $z_p \rightarrow \infty$.

The total energy E_t as a minimum of parameters A and B was calculated by the Monte Carlo method at different values of parameters D (separation between e and h layers) and $r_s = \frac{1}{\sqrt{\pi n}}$.

The results are given in table 1. The energy $E(D)$ and the density of the liquid state decreases as D grows. At $D = 0$ and $r_s \rightarrow \infty$ the energy equals the two-dimensional exciton energy $\text{Ryd}_2^h * = \frac{2\mu e^4}{\hbar^2 \epsilon}$. At large r_s the calculated function $E(D)$ agrees with the energy of the two-dimensional spatially indirect exciton [31]. As results of calculations show, the gap Δ appears in the spectrum of new quasiparticles, increasing with the decrease of density n (see figure 2). So the system (for finite z_p) is an insulator. If $n \rightarrow 0$ the gap becomes equal to the binding energy of the two-dimensional exciton.

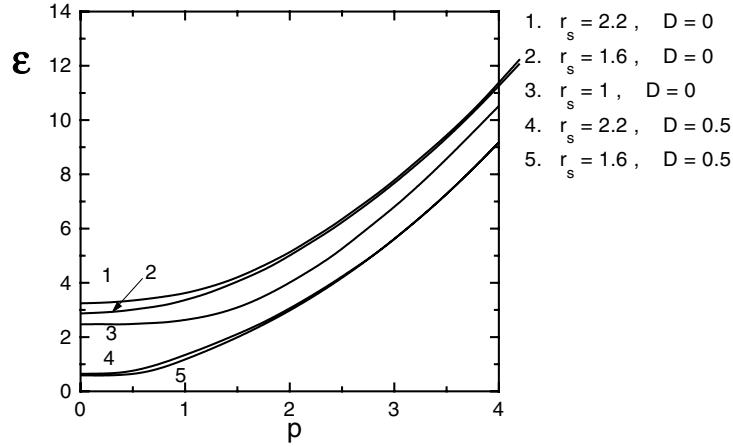


Figure 2. The spectrum of quasiparticles $E(p)$ (in units Ryd_2^* ; p is momentum in units p_F). 1— $r_s = 2.2$, $D = 0$; 2— $r_s = 1.6$, $D = 0$; 3— $r_s = 1.0$, $D = 0$; 4— $r_s = 2.2$, $D = 0.5$; 5— $r_s = 1.6$, $D = 0.5$.

Table 1. Isolated exciton energy E_{ex} , total energy of exciton liquid E_t , binding energy $E_b = E_{ex} - E_t$ of liquid phase, equilibrium distance between excitons in the liquid state r_s^{min} ($r_s^{min} = \frac{1}{a^* \sqrt{\pi n_{min}}}$; n_{min} is equilibrium density), Kosterlitz–Thouless superfluid transition and BCS critical temperatures as functions of interlayer separation D . Here μ is effective mass; ϵ is the dielectric constant. Values T_c and T_c^0 in the first three lines correspond to equilibrium densities of liquid phase at corresponding distance D ; the fourth line correspond to fixed $r_s = 3.6$ (liquid phase does not exist at $D \geq D_{cr} \approx 1.1$); $a^* = \frac{\hbar^2 \epsilon}{4\mu_{ex} e^2}$ and $\text{Ryd}_2^* = \frac{2\mu_{ex} \epsilon^4}{\hbar^2}$ are the radius and energy of the two-dimensional exciton (at $D = 0$).

D	$-E_{ex}$ (in Ryd_2^*)	$-E_t$ (in Ryd_2^*)	E_b (in Ryd_2^*)	r_s^{min}	$k_B T_c^0 \times 10^{-3}$ (in Ryd_2^*)	$k_B T_c \times 10^{-3}$ (in Ryd_2^*)
0	1.0	1.06	0.06	2.2	1.7	1.3
0.5	0.80	0.84	0.04	2.6	1.3	1.1
1.0	0.50	0.51	0.01	3.2	0.8	0.7
5.0	0.26	—	—	—	—	0.2

5. Rare system of excitons with spatially separated electrons and holes

The distinction between excitons and bosons manifests itself in exchange effects (see, e.g., [11, 24]). These effects for indirect excitons at large interwell separation D in a rare system $na^2(D) \ll 1$ are suppressed because of the negligible overlap of wavefunctions of two excitons due to the potential barrier, connected with the dipole–dipole repulsion of indirect excitons [11] (here $a(D)$ is the exciton radius along quantum wells). At large D the small parameter mentioned above for two indirect excitons with parallel dipoles has the form $T = \exp[-(D/2a^*)^{1/4}]$ ($a^* = \frac{\hbar^2 \epsilon}{4\mu_{ex} e^2}$; $\mu_{ex} = \frac{m_e m_h}{m_e + m_h}$ is the effective mass of the indirect exciton; m_e and m_h are masses of the electron and hole). The contribution of the exchange interaction to the chemical potential μ_{ex} at small densities n and small distances between layers D is

$$\mu_{ex} \sim T n a^2 E_0, \quad (16)$$

where $a = (8a^*)^{1/4} D^{3/4}$ is the radius of the indirect exciton [31]; E_0 is the binding energy of the indirect exciton.

So at large D the exchange interaction in the spatially separated system is suppressed in contrast to the e–h system in a single well due to the smallness of tunnel exponent

$T \sim \exp[-(D/2a^*)^{1/4}]$ connected with the penetration through the barrier of dipole–dipole interaction (see also [11]). Hence, at $D \gg a^*$, exchange phenomena, connected with the distinction between excitons and bosons, can be neglected.

As a result we obtain for the contribution of the exchange interaction μ_{ex} in chemical potential at small n and $D(D \ll a^*)$

$$\mu_{ex} = 4.71n(1 - D). \quad (17)$$

The contribution of the dipole interactions can be represented by the sum of the ladder diagrams given in figure 3. The Bogolubov approximation for a two-dimensional weakly interacting Bose gas is not valid due to the divergence of the two-dimensional amplitude of the scattering in the Born approximation [32]. For small transferred momenta $q \rightarrow 0$, essential for low densities n , the integrand in q behaves for the diagram in figure 3(a) as $\sim \frac{1}{q^3}$ and for the diagram in figure 3(b) as $\sim \frac{1}{q^2}$. Therefore, the diagrams in figure 3(a) are more important for the energy than that of figure 3(b). Therefore at $n \rightarrow 0$ we can take into account the direct dipole–dipole interaction of excitons in the framework of two-dimensional Bose gas theory in the ladder approximation [32]. The relation of vertex Γ to the two-dimensional scattering amplitude $f_0(\kappa)$ is

$$\Gamma = -2f_0(\kappa) \left(\frac{2\pi\kappa}{i} \right)^{\frac{1}{2}}, \quad (18)$$

and the contribution of the dipole–dipole interaction to the chemical potential is $\mu_d = \Gamma n = \frac{\kappa^2}{2}$; the two-dimensional scattering amplitude for the interaction potential in the form $U(r) = Ar^{-n}$ ($n > 2$) is $f_0(\kappa) = \frac{(\pi i/2\kappa)^{\frac{1}{2}}}{\ln(\kappa A^{1/n-2})}$.

At $\ln((\kappa A)^{-1}) \gg 1$ the contribution of the dipole–dipole interaction to the chemical potential is

$$\mu = \frac{8\pi n}{2M \ln \left(\frac{1}{(8\pi e^4 D^2 M^2 n)} \right)}, \quad (19)$$

where $M = m_e + m_h$ is the mass of the indirect exciton.

The estimation of the contribution of the van der Waals interaction to the chemical potential μ_v is analogous to the dipole–dipole one. As a result for the contribution of the van der Waals interaction the equations above are valid, where $n = 6$ and $A = C_6$ is the coefficient of the van der Waals interaction.

Now we shall find the coefficient C_6 for small and large interlayer separation D . The coefficient C_6 is related to the polarizability α of the ground state of the two-dimensional exciton (compare [33] for the three-dimensional case):

$$C_6 = -\frac{3}{2}\alpha\rho^2, \quad (20)$$

where ρ is the exciton radius.

The polarizability α is

$$\alpha = -2e^2 \sum_k \frac{|x_{0k}|^2}{E_0 - E_k}, \quad (21)$$

where x is the relative e–h coordinate of the exciton with spatially separated e and h along layers; E_k are exciton energies. The series (21) may be summed by using the auxiliary operator b (see [34]):

$$x = \frac{m}{\hbar} \frac{db}{dt}. \quad (22)$$

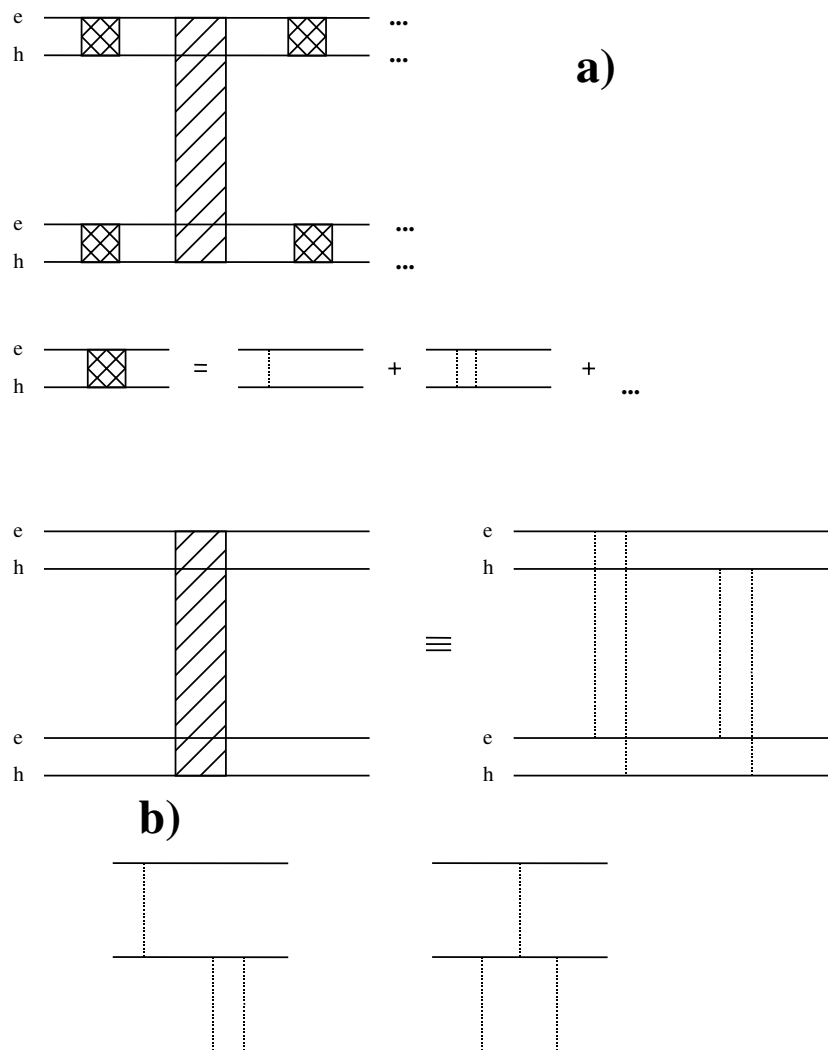


Figure 3. (a) Essential diagrams for the rare interacting exciton system; ladder diagrams for a single exciton; interaction between excitons ($\sim e^2 D^2 / r^3$ at $r \gg D$); part of the essential diagram; (b) omitted diagrams for $n \rightarrow 0$.

In a standard way one can introduce the function $f(r)$ through the equality $b = \cos \phi f(r)$. The function $f(r)$ obeys the equation

$$ir = \frac{1}{2} f'' + \frac{1}{2r} f' - \frac{1}{2r^2} f + \frac{\psi_0'}{\psi_0} f', \tag{23}$$

where ψ_0 is the wavefunction of the ground state for the exciton. The perturbed function $\psi_0(r)$ for small D is calculated in the first order of $U(r) = \frac{2e^2 D^2}{\epsilon r^3}$, the wavefunctions for the two-dimensional exciton at $D = 0$ being the zeroth order approximation. For large D the ground state of the exciton is characterized by the wavefunction of the oscillator with the radius [31] $a = (8a^*)^{1/4} D^{3/4}$.

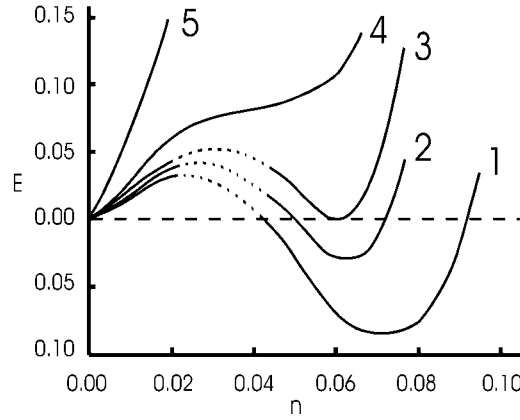


Figure 4. Energy (E) of the system of spatially separated e and h versus density n ; E and n are in units of binding energy $\text{Ryd}_2^* = \frac{2\mu_{ex}e^4}{\hbar^2\epsilon}$ and $(a^*)^{-3}$, where $a^* = \frac{\hbar^2\epsilon}{4\mu_{ex}e^2}$ is the effective Bohr radius of the two-dimensional exciton at $D = 0$; curves 1–5 correspond to the following values of D : 1— $D = 0$; 2— $D = 0.5$; 3— $D = 1.0$; 4— $D = 1.9$; 5— $D = 5.0$; limit $E = 0$ for all curves corresponds to (different) exciton energies $E_{ex}(D)$ for the corresponding D .

Using the solution of the differential equation for $f(r)$, we find eigenvalues of the operator b . On substituting them into equation (22), we find z . After summing the series equation (21) we obtain for the polarizability α at small D

$$\alpha = \frac{21}{16}a^3 \left(1 + \frac{64\pi^2 m^2 e^4 D^4}{\hbar^4 a^2} \right). \quad (24)$$

For large D we have

$$\alpha = 0.93 D^{\frac{9}{4}} a^{*\frac{3}{4}}. \quad (25)$$

On substituting equations (24) and (25) into (20), we find the contribution of the van der Waals interaction to the chemical potential.

As calculations show, already at $D = 0.5$ the contribution of the van der Waals interaction to the energy $E(n)$ and to the chemical potential is negligible ($\sim 10^{-2}$) in comparison with the contribution from the dipole–dipole interaction.

The exchange interaction in the spatially separated system for this region D is also smaller than that for the single-layer e–h system. This is due to small tunnelling through the barrier originated from the dipole–dipole interaction. The total chemical potential μ of the system is obtained as the sum of exchange, dipole–dipole and van der Waals contributions. The total energy for the rare exciton gas is found by the integration of relation $\mu = \frac{dE}{dn}$.

For crude estimation of the energy of the ground state at *all* surface densities n one may use an interpolation between results of the variational calculations at $n \geq 1$ (see section 4) and low densities (section 5).

6. Discussion of the results of calculations of liquid and gas states

The results of the variational calculation (section 4) show that in a single-layer (isotropic) e–h system ($D = 0$) the exciton *liquid* phase exists with the energy $E \cong -1.05E_{ex}$, where E_{ex} is the binding energy of the two-dimensional exciton, and the equilibrium distance between particles is $r \cong 2.2a^*$, where a^* is the effective Bohr radius of the two-dimensional exciton (these results agree with [12]).

On increasing the distance between layers D the binding energy of the liquid decreases, and the equilibrium separation between particles increases (see figure 4). At $D = 1.1a^*$ the energy of the liquid phase becomes equal to that of the isolated exciton, and at $D > 1.1a^*$ the liquid becomes metastable, the exciton gas phase being stable (figure 4). At $D \geq D_{sp} = 1.9a^*$ minima on the dependence $E(n)$ disappear, so the liquid phase becomes absolutely unstable. The gas–liquid quantum phase transition in the exciton system has first order (interwell separation D being the controlling parameter in the problem).

7. The Kosterlitz–Thouless transition

The variational calculation gives $\Delta \neq 0$ for all interlayer separations D and densities n for the *isotropic* e–h system (section 4). Besides the rare case corresponds (also for the anisotropic case) to a slightly interacting Bose gas of excitons (section 5). So the excitonic phase exists for the isotropic case in the whole n – D plane. The superfluidity appears in the two-dimensional Bose system below the Kosterlitz–Thouless transition temperature T_c [23]:

$$T_c = \frac{\pi n_s}{2M}, \quad (26)$$

where only coupled vortexes can exist.

At small momenta the collective spectrum of the rare exciton system is the sound like $\varepsilon(p) = c_s p$ ($c_s = \sqrt{\frac{\mu}{M}}$ is the sound velocity) and satisfies Landau criterion for superfluidity. The density of the superfluid component $n_s(T)$ for a two-dimensional system with the sound spectrum can be estimated as

$$n_s = n - \frac{3\zeta(3)}{2\pi} \frac{T^3}{c_s^4 M}. \quad (27)$$

The second term in equation (27) is the temperature dependent normal density taking into account the gas of phonons (‘bogolons’) with dispersion law $\varepsilon(p) = \sqrt{\mu/M} p$; μ is given by equation (19).

Substituting the estimate for the density of the superfluid component n_s from equation (27) to (26), we obtain the equation for temperature T_c of the Kosterlitz–Thouless transition. Its solution is

$$T_c = \left[\left(1 + \sqrt{\frac{16}{(6 \times 0.45)^3 \pi^4} \left(\frac{MT_c^0}{n} \right)^3 + 1} \right)^{1/3} + \left(1 - \sqrt{\frac{16}{(6 \times 0.45)^3 \pi^4} \left(\frac{MT_c^0}{n} \right)^3 + 1} \right)^{1/3} \right] \frac{T_c^0}{(4\pi)^{1/3}}. \quad (28)$$

Here T_c^0 is an auxiliary quantity equal to the vanishing temperature of the superfluid density in the mean field approximation $n_s(T_c^0) = 0$:

$$T_c^0 = \left(\frac{32}{3\zeta(3) \ln^2 \left(\frac{1}{8\pi n M^2 D^4} \right)} \right)^{1/3} \frac{\pi n}{M}. \quad (29)$$

We consider a system of *large and intermediate densities*. For the estimation of the contribution of one-particle excitations to the normal component at large and intermediate densities n we use the relation $n_s(T)$ with T_c^0 in the BCS approximation at $T_c^0 - T_c \ll T_c^0$. As a result the density of the superfluid component can be expressed in the BCS approximation through the total density and the critical temperature T_c^0 [35]:

$$\frac{n_s(T)}{n} = \frac{2(T_c^0 - T)}{T_c^0}. \quad (30)$$

In this approximation

$$\Delta(0) = 1.76T_c^0, \quad (31)$$

where $\Delta(0)$ is the gap in the excitonic phase considered above (see section 4).

It is necessary also to take into account the contribution of the collective excitations to the density of the normal component n_n . At small temperatures the contribution of elementary excitations to the density of the normal component can be estimated in the approximation of an almost ideal equilibrium Bose gas. As a result we have for the superfluid density $n_s = n - n_n$

$$n_s = n - n \frac{2T - T_c^0}{T_c^0} - \frac{3\zeta(3)}{2\pi} \frac{T^3}{c_s^4 M}, \quad (32)$$

where $c = \sqrt{\frac{n}{m} \frac{d\mu}{dn}}$ is the sound velocity of a low-density exciton gas. The chemical potential μ of the system is determined from the variational approach (see section 4).

As a result, if we substitute equations (27) or (32) into equation (26) we can obtain estimations for the critical temperatures of the Kosterlitz–Thouless transition (see table 1). The temperature of the Kosterlitz–Thouless transition is decreasing at the increase D . At $T = T_{KT}$ the vortex–antivortex pair dissociation in the exciton system takes place and the *global* superfluid density has a universal jump [23]. Local superfluid density fluctuations can be observed even at $T > T_c$ by local photoluminescence study (see also the conclusion).

8. Mott transition in an anisotropic e–h system

Above we consider the *isotropic* e–h system with identical Fermi surfaces for electrons and holes (circles with the same radii) at high and middle densities $n_e = n_h$. The ground state of the isotropic e–h system (with attractive e–h interaction) *at any density* is not stable due to e–h pairing [24], analogously to the Cooper instability in a one-component system [28].

In the *anisotropic* two-dimensional e–h system Mott metal–insulator transition takes place analogously to the three-dimensional anisotropic e–h system [26]. We shall find critical densities n_{cr} for the Mott transition as a function of interlayer separation D for a simple model of the e–h system. At $n \geq n_{cr}$ the ‘metal’ e–h phase becomes preferable in the e–h system; at $n < n_{cr}(D)$ the excitonic phase is the ground state of the system (at the same time the excitonic gas is always a metastable state due to dipole–dipole repulsion of excitons with spatially separated e and h).

We use for simplicity the model with congruent (parallel) regions of two-dimensional Fermi surfaces of electrons and holes with length κ_1 . For simplicity we take $\frac{\kappa_1}{p_F} \ll 1$. Let the distance between congruent regions of e and h Fermi surfaces be κ_2 . We suggest also that $\frac{\kappa_2}{p_F} \ll 1$.

We suppose that the gap in the spectrum of excitations is small ($\Delta \ll \epsilon_F$) near the Mott critical density, i.e. the Mott quantum phase transition has second order (this will be proved below). As usual we suppose that the value of the gap $\Delta(p)$ is independent of the momentum and equal to the constant Δ at $p < \kappa_1$, and equal to zero in other regions. The region of slow decrease of the gap is proportional to Δ as proved in [26]. As Δ is small near n_{cr} we neglect the region along the Fermi surfaces where the gap is decreasing.

Analogously to calculations in section 2 we obtain in the Hartree–Fock approximation (with e–h pairing taken into account)

$$E_{HF} = \left[\pi n \kappa_1^4 - 4\sqrt{\pi n} \left(\left(\sqrt{\kappa_1^2 + \kappa_2^2} - \kappa_2 \right) \frac{\kappa_1^2}{2} + \frac{\kappa_1^3}{3} \ln \frac{\sqrt{\kappa_1^2 + \kappa_2^2}}{\kappa_2} + \pi n \sqrt{2} D \kappa_1^4 \right) \right] \Delta^2. \quad (33)$$

It turns out that the correlation energy may be estimated in the RPA taking account of e–h pairing. The contribution of diagrams essential for large momenta is proportional to Δ^4 and so is negligible near $n = n_{cr}$. As a result using equation (8) we have for the correlation energy E_{corr}

$$E_{corr} = 64 \frac{\kappa_2}{p_F} \left(1 + \frac{\kappa_2 D}{p_F} \right) \Delta^2. \quad (34)$$

Contributions of order Δ^4 in $E_{corr} = A\Delta^4 + B\Delta^2 + C$ do not affect the critical Mott concentration obtained from the condition $\Delta_m = 0$ where $\Delta = \Delta_m$ is calculated from the minimization of the total energy (only the inequality $A \neq 0$ for coefficient A in E_{corr} is essential). The total energy E is

$$E = E_{HF} + E_{corr}. \quad (35)$$

Minimizing the total energy E on the order parameter Δ , we obtain (at $A \neq 0$) the gap Δ as a function of the density n and of interlayer separation D :

$$\Delta^2 = \frac{1}{128\kappa_2^2} \left[(1 + \sqrt{2}D)\pi n \kappa_1^4 - 4\sqrt{\pi n} \left(\left(\sqrt{\kappa_1^2 + \kappa_2^2} - \kappa_2 \right) \frac{\kappa_1^2}{2} + \frac{\kappa_1^3}{3} \ln \frac{\sqrt{\kappa_1^2 + \kappa_2^2}}{\kappa_2} \right) + 64\kappa_2 \right]. \quad (36)$$

The result, equation (36), does not depend on the value of coefficient A . From the condition $\Delta = 0$ in equation (36) we find the critical density n_{cr} of the metal–insulator transition as a function of D :

$$n_{cr}(D) = \frac{1}{(1 + \sqrt{2}D)\pi \kappa_1^4} \left[2 \left(\left(\sqrt{\kappa_1^2 + \kappa_2^2} - \kappa_2 \right) \frac{\kappa_1^2}{2} + \frac{\kappa_1^3}{3} \ln \frac{\sqrt{\kappa_1^2 + \kappa_2^2}}{\kappa_2} \right) + \sqrt{4 \left(\left(\sqrt{\kappa_1^2 + \kappa_2^2} - \kappa_2 \right) \frac{\kappa_1^2}{2} + \frac{\kappa_1^3}{3} \ln \frac{\sqrt{\kappa_1^2 + \kappa_2^2}}{\kappa_2} \right)^2 - 64\kappa_2(1 + \sqrt{2}D)\pi \kappa_1^4} \right]. \quad (37)$$

As follows from equations (36) and (37), at $n \rightarrow n_{cr}$ we have $\Delta \rightarrow 0$, i.e. the Mott quantum phase transition has second order.

9. Indirect biexcitons in the superlattice slab

Now we consider an isotropic slab of the superlattice with alternating e and h layers. We show that the rare system of weakly interacting two-dimensional indirect excitons in this superlattice is unstable, in contrast to the two-layer system in CQWs. At small densities $na^2 \ll 1$ the system of indirect excitons at low temperatures is the two-dimensional weakly nonideal Bose gas with dipole moments d normal to the wells in the ground state ($d \sim eD$, D is the interwell separation), increasing with the distance between wells D ($a(D)$ is the radius of the excitons along the wells; $a \approx a^* = \hbar^2/\mu_{ex}e^2$ at $D \ll a^*$ and $a \approx a^{*\frac{1}{4}}D^{\frac{3}{4}}$ at $D \gg a^*$, when the spectrum of low lying levels is equivalent to that of the two-dimensional oscillator; $\mu_{ex} = \frac{m_e m_h}{m_e + m_h}$ is the effective mass of the indirect exciton). In contrast to ordinary excitons, for spatially indirect excitons the main contribution to the energy originates from dipole–dipole interactions U_- and U_+ of excitons with opposite and parallel dipoles, respectively. Two parallel (+) and opposite (–) dipoles in the rare system interact as $U_+ = -U_- = \frac{e^2 D^2}{\epsilon R^3}$, where ϵ is the dielectric constant; R is the distance between dipoles along quantum well planes; we

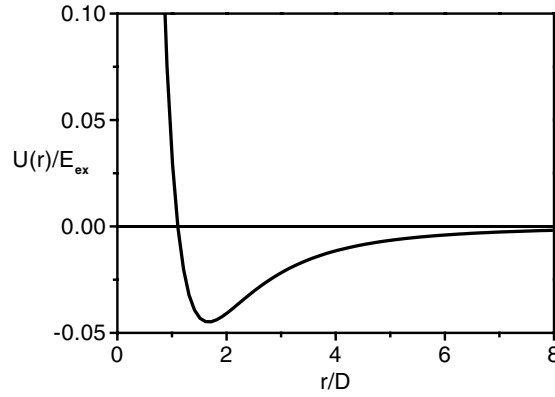


Figure 5. The potential energy $U(r)$ of the interaction of indirect excitons with opposite dipoles, located in neighbouring pairs of wells (in units of the binding energy of the indirect exciton $E_{ex} = e^2/\epsilon D$), as a function of the distances r between excitons along the wells (in units of D).

suppose that $D/R \ll 1$ and $L/R \ll 1$ (L is the mean distance between dipoles in the direction normal to the well planes). Below we consider the case when the number of quantum wells k in the superlattice is restricted, $k \ll \frac{1}{D\sqrt{\pi n}}$, and this is valid for small k or for very low exciton density (n is the exciton surface density). In this case the inequality $L/R \ll 1$ is valid.

At large D for two indirect excitons with opposite dipoles the small tunnelling parameter is $\exp[-0.33\sqrt{2D}/a^*]$ due to Coulomb repulsion at small distances (see figure 5). In section 2 it was shown that exchange effects in the rare system of indirect excitons can be neglected.

The total Hamiltonian \hat{H}_{tot} of the rare system of indirect excitons in the superlattice is

$$\hat{H}_{tot} = \hat{H}_0 + \hat{H}_{int}. \quad (38)$$

Here \hat{H}_0 is the Hamiltonian of the system of noninteracting excitons:

$$\hat{H}_0 = \sum_p \varepsilon_0(p)(a_p^+ a_p + b_p^+ b_p + a_{-p}^+ a_{-p} + b_{-p}^+ b_{-p}), \quad (39)$$

where $\varepsilon_0(p) = \frac{p^2}{2M}$ is the spectrum of an isolated two-dimensional indirect exciton ($M = m_e + m_h$, m_e and m_h are the exciton, electron and hole mass, respectively). a_p^+ , a_p , b_p^+ and b_p are the creation and annihilation operators of excitons with up and down dipoles; \hat{H}_{int} is the Hamiltonian of the interaction between excitons:

$$\begin{aligned} \hat{H}_{int} = \frac{U}{2S} \sum_{p_1+p_2=p_3+p_4} & (a_{p_4}^+ a_{p_3}^+ a_{p_2} a_{p_1} + b_{p_4}^+ b_{p_3}^+ b_{p_2} b_{p_1} - a_{p_4}^+ a_{p_3}^+ b_{p_2} b_{p_1} - b_{p_4}^+ b_{p_3}^+ a_{p_2} a_{p_1} \\ & - a_{p_4}^+ b_{p_3}^+ a_{p_2} b_{p_1}); \end{aligned} \quad (40)$$

S is the surface of the system. Let us consider the temperature $T = 0$. Assuming the majority of particles are in the condensate ($(N - N_0)/N_0 \ll 1$, where N and N_0 are the total number of particles and the number of particles in the condensate), we account as in the Bogolyubov approximation only for interaction between condensate particles and of excited particles with condensate particles, neglecting the interaction between overcondensate particles. Then the total Hamiltonian transforms to

$$\begin{aligned} \hat{H}_{tot} = \frac{1}{2} \sum_{p \neq 0} [\varepsilon_0(p)(a_p^+ a_p + b_p^+ b_p + a_{-p}^+ a_{-p} + b_{-p}^+ b_{-p}) - Un(a_p^+ b_{-p}^+ + a_p b_{-p} + a_{-p}^+ b_p^+ + a_{-p} b_p \\ + a_p^+ b_p + a_{-p}^+ b_{-p} + a_p b_p^+ + a_{-p} b_{-p}^+)]. \end{aligned} \quad (41)$$

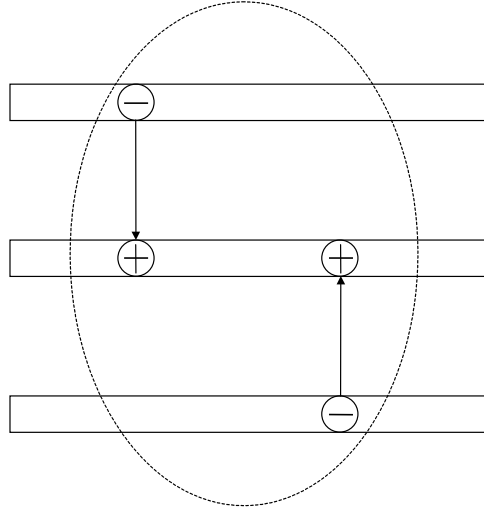


Figure 6. Two-dimensional indirect biexcitons consisting of indirect excitons with opposite dipole moments, located in neighbouring pairs of wells.

In equation (41) terms, arising from first and second terms of the Hamiltonian equation (40), which describe the repulsion of the indirect excitons with parallel dipole moments, are compensated by other terms of the Hamiltonian equation (40), describing the attraction of indirect excitons with opposite dipoles. In the result only terms describing the attraction survive. Let us diagonalize Hamiltonian \hat{H}_{tot} by using the unitary transformation of the Bogolyubov type [27]

$$\begin{aligned} a_p &= \frac{1}{\sqrt{1 - A_p^2 - B_p^2}} (\alpha_p + A_p \alpha_{-p}^+ + B_p \beta_{-p}^+); \\ b_p &= \frac{1}{\sqrt{1 - A_p^2 - B_p^2}} (\beta_p + A_p \beta_{-p}^+ + B_p \alpha_{-p}^+), \end{aligned} \quad (42)$$

where the coefficients A_p and B_p are found from the condition of vanishing of coefficients at nondiagonal terms in the Hamiltonian. As a result we obtain

$$\hat{H}_{tot} = \sum_{p \neq 0} \varepsilon(p) (\alpha_p^+ \alpha_p + \beta_p^+ \beta_p) \quad (43)$$

with the spectrum of quasiparticles $\varepsilon(p)$:

$$\varepsilon(p) = \sqrt{(\varepsilon_0(p))^2 - (nU)^2}. \quad (44)$$

At small momenta $p < \sqrt{2MnU}$ the spectrum of excitations becomes imaginary. Hence, the system of weakly interacting indirect excitons in a slab of a superlattice is unstable.

This, on first view, strange result can be illustrated by the following example. There are equal numbers of dipoles oriented up and down. Let us consider four dipoles, two of them being oriented up and two down. It is easy to count directly that the number of repelling pairs is smaller than that of attracting ones. The prevailing attraction leads to instability.

Let us consider as the ground state of the system the rare weakly nonideal gas of two-dimensional indirect biexcitons, created by indirect excitons with opposite dipoles in neighbouring pairs of wells (figure 6). In spite of the fact that masses of electrons and holes

are of the same order for the biexciton the adiabatic approximation turns out to be applicable. The small parameter is not the ratio between masses of the electron and hole, but the numerical small parameter is equal to the ratio of biexciton and exciton energies or the ratio between radii of the biexciton and exciton along quantum wells (see, e.g., [36]). These parameters are small at $m_e = m_h$, and they are even smaller than analogous parameters for atoms and molecules. The smallness of these parameters will be verified below by the results of the calculation of the indirect biexciton. Here, for simplicity, it is assumed that the distance between wells D is greater than the radius of the indirect biexciton a_0 : $D \gg a_0 \sim a^* \frac{1}{4} D^{\frac{3}{4}}$. The potential energy of interaction between indirect excitons with opposite dipoles $U(r)$ has the form shown in figure 5 (r is the distance between indirect excitons along quantum wells):

$$U(r) = \frac{e^2}{\epsilon r} - \frac{2e^2}{\epsilon \sqrt{r^2 + D^2}} + \frac{e^2}{\epsilon \sqrt{r^2 + 4D^2}}, \quad (45)$$

where ϵ is the dielectric constant. At $r > 1.11D$ indirect excitons attract, and at $r < 1.11D$ they repel. The minimum of potential energy $U(r)$ locates at $r = r_0 \approx 1.67D$ between indirect excitons. At large D one can expand the potential energy $U(r)$ on the parameter $\frac{r-r_0}{D} \ll 1$ up to the second order:

$$U(r) = -0.04 \frac{e^2}{\epsilon D} + 0.44 \frac{e^2}{\epsilon D^3} (r - r_0)^2. \quad (46)$$

So at large D biexciton levels correspond to the two-dimensional harmonic oscillator with the frequency $\omega = \sqrt{\frac{0.88e^2}{M\epsilon D^3}}$:

$$E_n = -0.04 \frac{e^2}{\epsilon D} + 2\sqrt{2}E_0 \left(\frac{r^*}{D}\right)^{3/2} (n+1), \quad (47)$$

where $E_0 = \frac{\mu_{ex} M e^4}{\hbar^2 \epsilon}$, $r^* = \frac{1}{2} a^*$, $e^2 = 0.88e^2$. In the ground state the characteristic spread of biexciton a_b along quantum wells (near the mean radius of biexciton r_0 along wells) is

$$a_b = \sqrt{\frac{2\hbar}{M\omega}} = (8r^*)^{1/4} D^{3/4} = 1.03a, \quad (48)$$

where $a = (8a^*)^{1/4} D^{3/4}$ is the radius of the indirect exciton.

Hence, the ratio of the binding energies of the biexciton and exciton is $E_{bex}/E_{ex} = 0.04 \ll 1$ at $D \gg a$ (the ratio of radii of the exciton and biexciton is $a/r_0 = 0.67(8a^*)^{1/4} D^{-1/4} \ll 1$). So the adiabatic condition is valid.

The mean dipole moment of the indirect biexciton is equal to zero. However, the quadrupole moment is nonzero and equal to $Q = 3eD^2$ (the large axis of the quadrupole is normal to the quantum wells). So indirect biexcitons interact at long distances $R \gg D$ as parallel quadrupoles: $U(R) = 9e^2 D^4 / R^5$. Exchange effects, connected with the distinction between rare indirect biexcitons and bosons, can be suppressed due to the negligible overlap of wavefunctions of two biexcitons on account of the potential barrier, associated with the quadrupole repulsion of indirect biexcitons at long distances analogously to dipole excitons. At large D the small tunnelling parameter connected with this barrier has the form $\exp[-0.93\sqrt{D/a^*}]$. Hence, at $D \gg a^*$ exchange effects for indirect biexcitons can be neglected.

We account for the scattering of biexciton on biexciton by using the results of the theory of the two-dimensional Bose gas [32]. The chemical potential μ of two-dimensional biexcitons, repulsed by the quadrupole law, in the ladder approximation, has the form ($\hbar = 1$) (compare equation (19))

$$\mu = \frac{4\pi n_{bex}}{M_b \ln \left(\frac{1}{8\pi(9e^2 D^4 M_b)^{2/3} n_{bex}} \right)}, \quad (49)$$

where $n_{bex} = \frac{1}{2}n$ is the density of biexcitons; $M_b = 2M$ is the mass of a biexciton.

For the estimation of the superfluidity and Kosterlitz–Thouless transition for the indirect biexcitons in superlattices equations (27)–(29) are valid with the substitution n_{bex} instead of n and M_b instead of M .

10. Conclusion

The exciton *liquid*–exciton *gas* quantum phase transition has been investigated in the isotropic system of spatially separated electrons and holes for increase of separation D between e and h layers. The binding energy of the exciton liquid is a decreasing function of D . At $D < D_m = 1.1a^*$ the energy of the liquid phase is greater than the energy of a single exciton, so the exciton liquid is a stable phase in this region and the exciton gas is a metastable phase (all distances are given in units of the two-dimensional exciton at $D = 0$). At $D = D_m = 1.1a^*$ the exciton liquid–exciton gas first order quantum transition takes place. The minimum on the curve $E(n)$ corresponding to the exciton *liquid* phase survives in the region $D_m < D < D_{cr}$ where $D_{cr} \approx 1.9a^*$, i.e. the liquid is metastable at these interlayer separations. At $D > 1.9a^*$ the liquid phase becomes absolutely unstable and the exciton density is determined only by external conditions. Excitonic gas is metastable at any D due to dipole–dipole repulsion of excitons with spatially separated e and h. At finite densities (smaller than the density of excitonic liquid n_{liq}) biexcitons ('H₂') and biexciton ions ('H₂⁺') and other complexes (particularly charged ones) with spatially separated electrons and holes also exist at small D (some of them become stabilized in strong magnetic fields). They also may have interesting transport properties along layers.

The system of excitons with spatially separated electrons and holes is superfluid at temperatures $T < T_{KT}(D)$ where $T_{KT}(D)$ is the Kosterlitz–Thouless transition critical temperature, $T_{KT}(D)$ decreasing with the separation between e and h layers.

For an anisotropic e–h system the metal–insulator Mott quantum phase transition is investigated and the critical densities of excitons at the Mott transition as functions of interlayer separation are studied.

Note that the liquid exciton phase can manifest itself, e.g. through the appearance of liquid drops of excitons, which can be found experimentally due to the motion of drops as gigantic fluctuations of photoluminescence at the local observation of photoluminescence using a nontransparent mask with a microhole or using an optical fibre. The observation of exciton drops through fluctuations of the current along the layers in CQWs is also possible. The interlayer resistance relating to drag of electrons and holes can also be a sensitive indicator of the gas–liquid phase transition and the transition to the superfluid and to other phases of the e–h system (see, e.g., [37, 38] and references therein). Linear and nonlinear optical properties also have some peculiarities in the system with e–h pairing (see, e.g., [39, 40]).

The collective spectrum, the density of the superfluid component and the Kosterlitz–Thouless transition temperature for the system of indirect excitons were calculated.

It is shown that the rare system of indirect excitons in a slab of a superlattice with alternating e and h layers turns out to be *unstable* due to the attraction of excitons with opposite dipoles at large distances. The rare system of indirect biexcitons in superlattices repelled as parallel quadrupoles is *stable*. So if the laser pumping increases at low temperatures the excitonic line can vanish and only biexcitonic line survive. At large D the biexciton is equivalent to a two-dimensional oscillator. Two biexcitons interact as parallel quadrupoles. The Kosterlitz–Thouless transition to the superfluid state is calculated for the system of indirect biexcitons.

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References

- [1] Fukuzawa T, Mendez E E and Hong J M 1990 *Phys. Rev. Lett.* **64** 3066
Kash J A, Zachau M, Mendez E E, Hong J M and Fukuzawa T 1991 *Phys. Rev. Lett.* **66** 2247
- [2] Sivan U, Solomon P M and Strikman H 1992 *Phys. Rev. Lett.* **68** 1196
- [3] Butov L V, Gossard A C and Chemla D S 2002 *Nature* **418** 751
Lozovik Yu E, Ovchinnikov I V, Volkov S Y, Butov L V and Chemla D S 2002 *Phys. Rev. B* **65** 235304
Butov L V, Zrenner A, Abstreiter G, Bohm G and Weimann G 1994 *Phys. Rev. Lett.* **73** 304
- [4] Cheng J P, Kono J, McCombe B D, Lo I, Mitchel W C and Stutz C E 1995 *Phys. Rev. Lett.* **74** 450
Snoko D, Denev S, Liu Y, Pfeiffer L and West K 2002 *Nature* **418** 754
- [5] Larinov A V, Timofeev V B, Ni P A, Dubonos S V, Hvam I and Soerensen K 2002 *JETP Lett.* **75** 570
Bayer M, Timofeev V B, Faller F, Gutbrod T and Forchel A 1996 *Phys. Rev. B* **54** 8799
- [6] Lozovik Yu E and Yudson V I 1975 *Pis. Zh. Eksp. Teor. Fiz.* **22** 26 (Engl. transl. 1975 *JETP Lett.* **22** 26)
Lozovik Yu E and Yudson V I 1976 *Zh. Eksp. Teor. Fiz.* **71** 738 (Engl. transl. 1976 *Sov. Phys.–JETP* **44** 389)
Lozovik Yu E and Yudson V I 1976 *Solid State Commun.* **18** 628
Lozovik Yu E and Yudson V I 1977 *Solid State Commun.* **21** 211
- [7] Zhu Xu, Littlewood P B, Hybertsen M S and Rice T M 1995 *Phys. Rev. Lett.* **74** 1633
- [8] Conti S, Vignale G and MacDonald A H 1998 *Phys. Rev. B* **57** R 6846
- [9] Naveh Y and Laikhtman B 1996 *Phys. Rev. Lett.* **77** 900
- [10] Liu L, Swerkowski L, Neilson D and Szymanski J 1996 *Phys. Rev. B* **53** 7923
- [11] Lozovik Yu E and Berman O L 1996 *Pis. Zh. Eksp. Teor. Fiz.* **64** 526 (Engl. transl. 1996 *JETP Lett.* **64** 573)
- [12] Klyuchnik A V and Lozovik Yu E 1979 *Zh. Eksp. Teor. Fiz.* **76** 670 (Engl. transl. 1979 *Sov. Phys.–JETP* **49** 335)
Lozovik Yu E and Poushnov A V 1997 *Phys. Lett. A* **228** 399
- [13] Shevchenko S I 1994 *Phys. Rev. Lett.* **72** 3242
- [14] Das Sarma S, Sachdev S and Zheng L 1997 *Phys. Rev. Lett.* **79** 917
- [15] Lerner I V and Lozovik Yu E 1978 *Zh. Eksp. Teor. Fiz.* **74** 274 (Engl. transl. 1978 *Sov. Phys.–JETP* **47** 146)
Lerner I V and Lozovik Yu E 1979 *Zh. Eksp. Teor. Fiz.* **76** 1136 (Engl. transl. 1979 *Sov. Phys.–JETP* **49** 576)
Lerner I V and Lozovik Yu E 1980 *Zh. Eksp. Teor. Fiz.* **78** 1167 (Engl. transl. 1980 *Sov. Phys.–JETP* **51** 588)
Lerner I V and Lozovik Yu E 1981 *Zh. Eksp. Teor. Fiz.* **80** 1488 (Engl. transl. 1981 *Sov. Phys.–JETP* **53** 763)
Dzjubenko A B and Lozovik Yu E 1983 *Fiz. Tverd. Tela* **25** 1519 (Engl. transl. 1983 *Sov. Phys.–Solid State* **25** 874)
Dzjubenko A B and Lozovik Yu E 1984 *Fiz. Tverd. Tela* **26** 1540 (Engl. transl. 1984 *Sov. Phys.–Solid State* **26** 938)
Dzjubenko A B and Lozovik Yu E 1991 *J. Phys. A: Math. Gen.* **24** 415
- [16] Paquet D, Rice T M and Ueda K 1985 *Phys. Rev. B* **32** 5208
- [17] Kallin C and Halperin B I 1984 *Phys. Rev. B* **30** 5655
Kallin C and Halperin B I 1985 *Phys. Rev. B* **31** 3635
- [18] Chen X M and Quinn J J 1991 *Phys. Rev. Lett.* **67** 895
- [19] Chemla D S, Stark J B and Knox W H 1993 *Ultrafast Phenomena VIII* vol 21, ed J-L Martin *et al* (Berlin: Springer)
- [20] Yoshioka D and MacDonald A H 1990 *J. Phys. Soc. Japan* **59** 4211
- [21] Lozovik Yu E, Berman O L and Tsvetus V G 1999 *Phys. Rev. B* **59** 5627
- [22] Filin A I, Timofeev V B, Gubarev S I, Birkedal D and Hvam J M 1997 *Pis. Zh. Eksp. Teor. Fiz.* **65** 623
- [23] Kosterlitz J M and Thouless D J 1973 *J. Phys. C: Solid State Phys.* **6** 1181
Nelson D R and Kosterlitz J M 1977 *Phys. Rev. Lett.* **39** 1201
- [24] Keldysh L V and Kozlov A N 1968 *Zh. Eksp. Teor. Fiz.* **54** 978 (Engl. transl. 1968 *Sov. Phys.–JETP* **27** 521)
see also
Keldysh L V and Kopaev Y V 1964 *Fiz. Tverd. Tela* **6** 2791 (Engl. transl. 1965 *Sov. Phys.–Solid State* **6** 2219)
Kozlov A N and Maksimov L A 1965 *Zh. Eksp. Teor. Fiz.* **48** 1184 (Engl. transl. 1965 *Sov. Phys.–JETP* **21** 790)

- Keldysh L V and Silin A P 1975 *Krat. Soob. FIAN (Russ.)* **33**
see also review:
Halperin B I and Rice T M 1968 *Solid State Phys.* **21** 115
- [25] Klyuchnik A V and Lozovik Yu E 1978 *Fiz. Tverd. Tela* **20** 625 (Engl. transl. 1978 *Sov. Phys.–Solid State* **20** 364)
- [26] The condition of e–h pairing with total momenta equal to zero at different e and h surfaces has the form $v_F \delta p < \Delta$. The physical sense of this inequality is the following: $v_F \delta p$ is the ‘loss’ of one-particle energy needed to reduce the electron and hole to the same momenta; Δ is the ‘gain’ due to e–h pairing.
Lozovik Yu E and Yudson V I 1975 *Fiz. Tverd. Tela* **17** 1613 (Engl. transl. 1975 *Sov. Phys.–Solid State* **17** 1613)
- [27] Abrikosov A A, Gor’kov L P and Dzyaloshinskii I E 1963 *Methods of Quantum Field Theory in Statistical Physics* (Englewood Cliffs, NJ: Prentice-Hall)
- [28] Schrieffer J R 1964 *Theory of Superconductivity* (New York: Benjamin)
- [29] Pines D and Nozieres P 1966 *The Theory of Quantum Liquids* (New York: Benjamin)
- [30] Brinkman W F and Rice T M 1973 *Phys. Rev. B* **7** 1508
- [31] Lozovik Yu E and Nishanov V N 1976 *Fiz. Tverd. Tela* **18** 3267 (Engl. transl. 1976 *Sov. Phys.–Solid State* **18** 1905)
- [32] Lozovik Yu E and Yudson V I 1978 *Physica A* **93** 493
- [33] Barash Yu S 1988 *Van der Waals Forces* (Moscow: Nauka)
- [34] Landau L D and Lifshits E M 1989 *Quantum Mechanics* (Moscow: Nauka)
- [35] Abrikosov A A 1987 *Foundations of the Metal Theory* (Moscow: Nauka)
- [36] Ivanov L N, Lozovik Yu E and Musin D R 1978 *J. Phys. C: Solid State Phys.* **11** 2527
- [37] Vignale G and MacDonald A H 1996 *Phys. Rev. Lett.* **76** 2786
- [38] Lozovik Yu E and Nikitkov M V 1997 *Zh. Eksp. Teor. Fiz.* **111** 1107 (Engl. transl. 1997 *Sov. Phys.–JETP* **84** 612)
- [39] Snoke D W, Wolfe J P and Mysyrowicz A 1990 *Phys. Rev. Lett.* **64** 2543
- [40] Lozovik Yu E and Poushnov A V 1998 *Phys. Rev. B* **58** 6608
Lozovik Yu E and Ovchinnikov I V 2002 *Phys. Rev. B* **66** 075124